Homework 1

Zezhou Li

10405265

**Problem 1:**

**1.** Use Matlab to plot the likelihood function for each class.

Codes:

|  |
| --- |
| x = -6:0.1:10;  norm\_w1 = normpdf(x,0,1);  plot(x,norm\_w1);  hold on  norm\_w2 = normpdf(x,2,2);  plot(x,norm\_w2);  title('Likelihood function for each class');  xlabel('x'); ylabel('p(x|wi)');  legend({'w1','w2'},'FontSize',16'); |

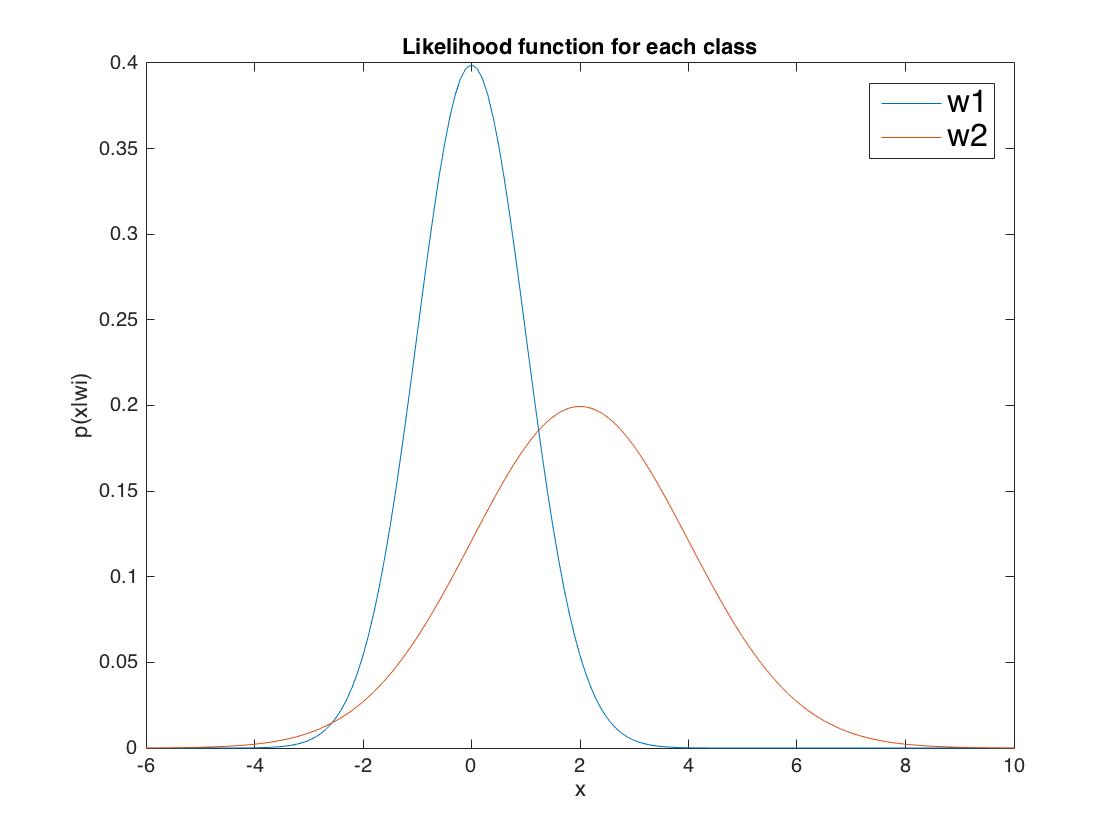
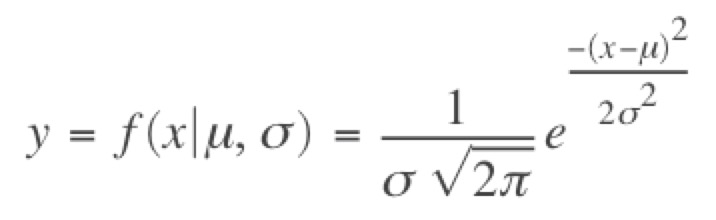


Figure 1. Likelihood function of each class

For normal distribution,



Thus, ,

Then when x=5,

p(x|w1)=

p(x|w2)=

**2.** p(x)= = =

Use Matlab to plot the evidence distribution:

Codes:

|  |
| --- |
| x = -10:0.1:10;  y = 3/5\*(1/sqrt(2\*pi)\*exp(-x.^2./2))+2/5\*(1/(2\*sqrt(2\*pi))\*exp(-(x-2).^2/8));  plot(x,y);  title('Evidence Distribution');  xlabel('x'); ylabel('p(x)'); |

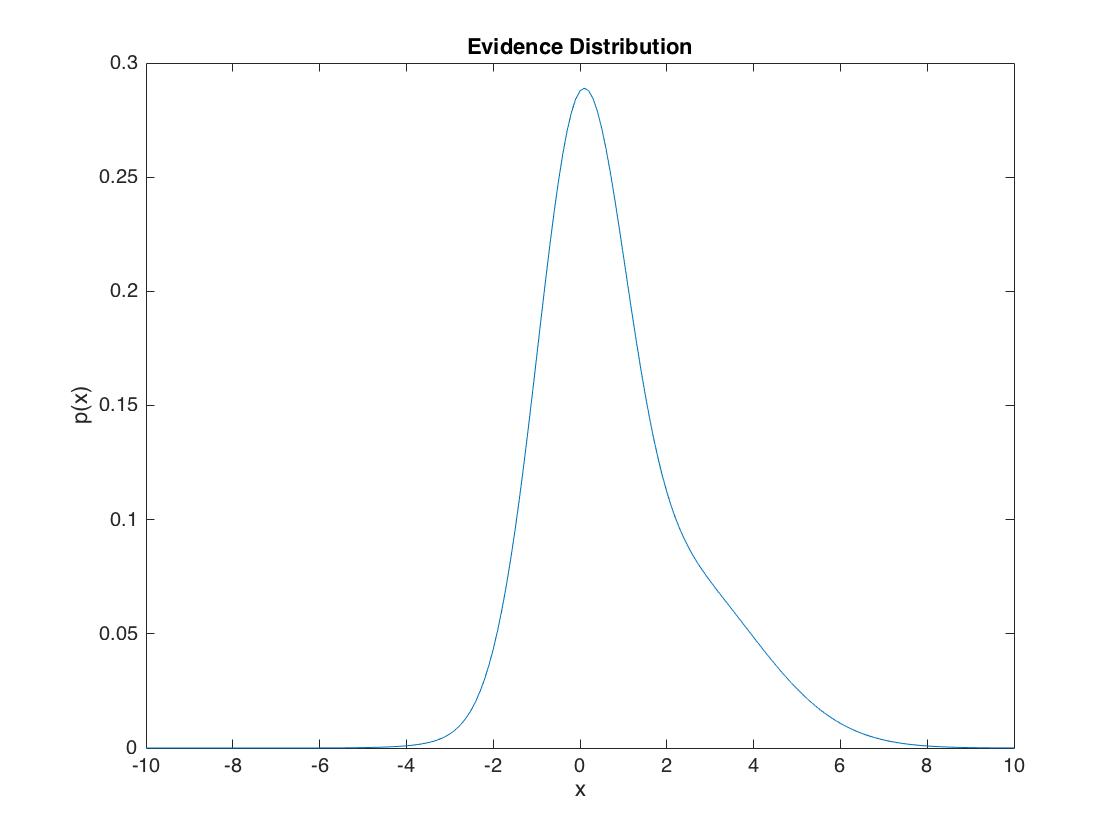


Figure 2. Evidence Distribution

When x=5, .

**3.** .

Thus, ,

Use Matlab to plot the posterior probability function for each class.

Codes:

|  |
| --- |
| x = -10:0.1:10;  norm\_w1 = normpdf(x,0,1);  norm\_w2 = normpdf(x,2,2);  norm\_x = 3/5\*norm\_w1+2/5\*norm\_w2;  norm\_post\_1 = 3/5.\*norm\_w1./norm\_x;  norm\_post\_2 = 2/5.\*norm\_w2./norm\_x;  plot(x,norm\_post\_1);  hold on  plot(x,norm\_post\_2);  title('Posterior probability function for each class');  xlabel('x'); ylabel('p(wi|x)');  legend({'P(w1|x)','P(w2|x)'},'FontSize',16'); |

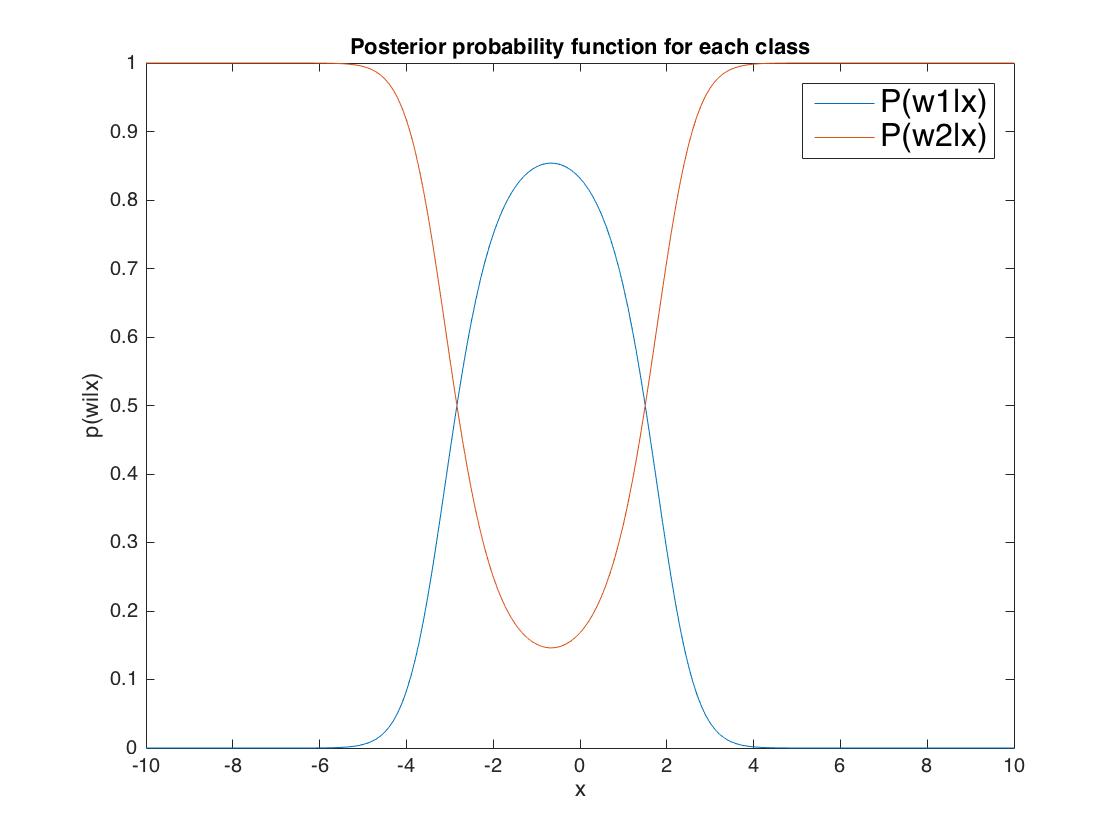


Figure 3. Posterior probability function for each class

When x=5, P(w1|x) = , P(w2|x) = 1.

**4.** The likelihood ratio function = ,

Use Matlab to plot the likelihood ratio function:

Codes:

|  |
| --- |
| x = -10:0.1:10;  norm\_w1 = normpdf(x,0,1);  norm\_w2 = normpdf(x,2,2);  plot(x,norm\_w1./norm\_w2);  title('Likelihood ratio function');  xlabel('x'); ylabel('p(x|w1)/p(x|w2)'); |

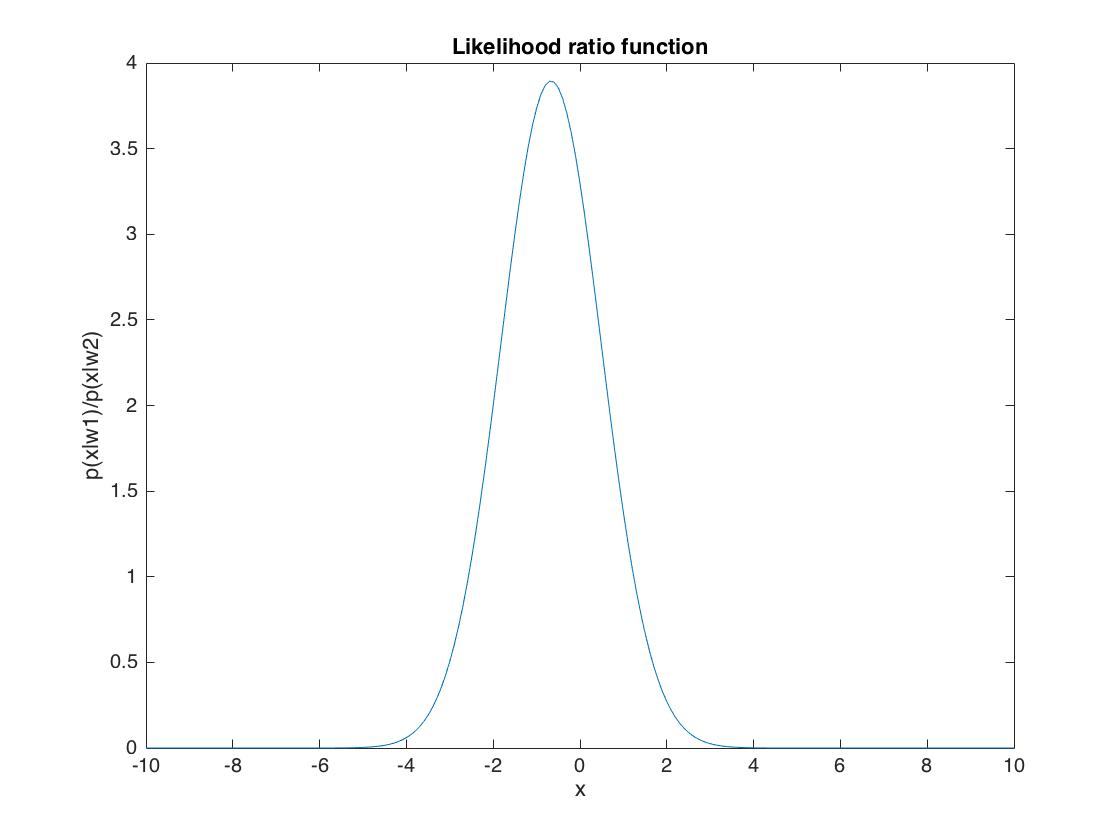


Figure 4. Likelihood ratio function

When x=5, likelihood ratio = .

**5.** For zero-one loss function,

Thus,

**6.** Here given the following risk matrix,

|  |  |  |
| --- | --- | --- |
|  | ******1 | ******2 |
| ***a***1 | 0 | 4 |
| ***a***2 | 2 | 0 |

**7.** The conditional risk here is:

Thus,

Use Matlab to plot the Bayes risk as a function of x.

Codes:

|  |
| --- |
| x = -10:0.1:10;  th=4/3;  norm\_w1 = normpdf(x,0,1);  norm\_w2 = normpdf(x,2,2);  l\_ratio = norm\_w1./norm\_w2;  c1 = (l\_ratio>th);  c2 = (l\_ratio<=th);  norm\_x = 3/5\*norm\_w1+2/5\*norm\_w2;  norm\_post\_1 = c1.\*4\*2/5.\*norm\_w2./norm\_x;  norm\_post\_2 = c2.\*2\*3/5.\*norm\_w1./norm\_x;  plot(x,norm\_post\_1);  hold on  plot(x,norm\_post\_2);  title('Bayes Risk');  xlabel('x'); ylabel('R(wi|x)');  legend({'R(w1|x)','R(w2|x)'},'FontSize',16'); |

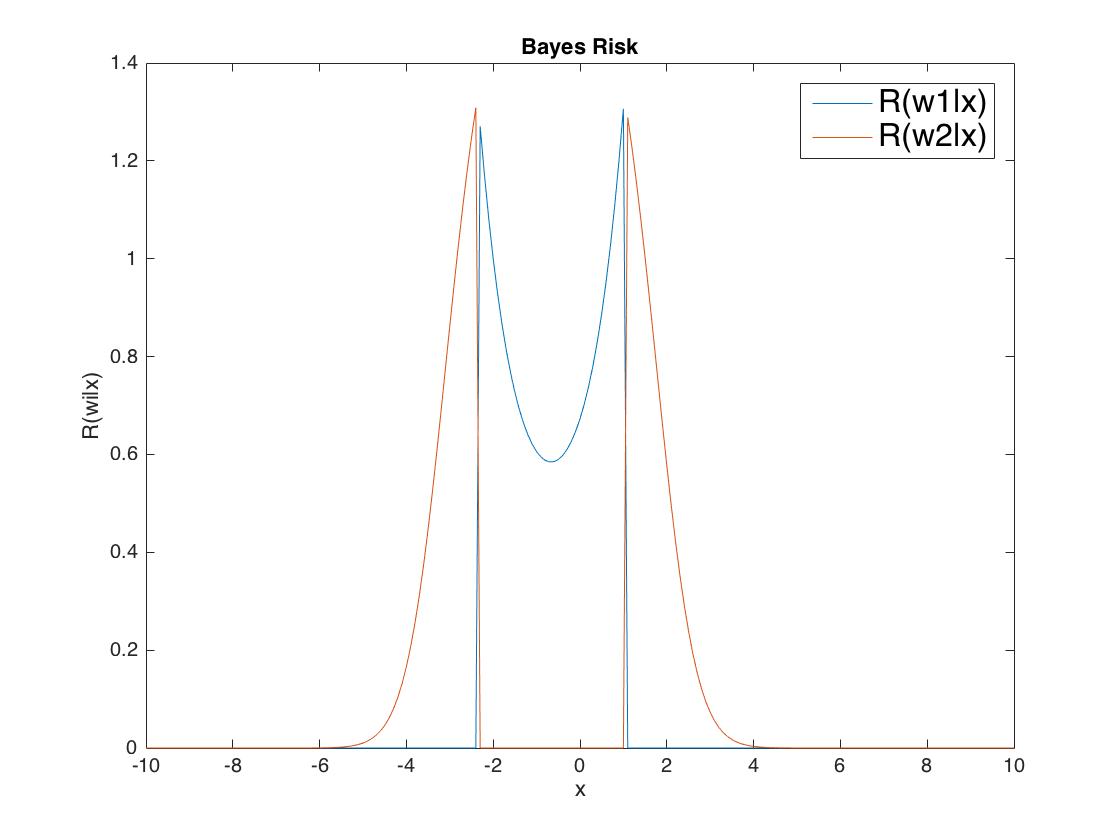


Figure 5. Bayes Risk as a function of x.

When x=5, , .

**Problem 2:**

**1.** Given the following estimated distribution parameters:

We can calculate : , .

Here assume P(w1)=0.25, P(w2)=0.75. Setting g1(x) = g2(x) to obtain the decision boundary.

We will then have:

Thus the decision boundary is:

**2.** We know that:

Thus, substitute the means and covariances into this equation, we can get:

Thus,